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SOLVED ASSIGNMENT

BCS-054

Computer Oriented Numerical Techniques

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STEP 1

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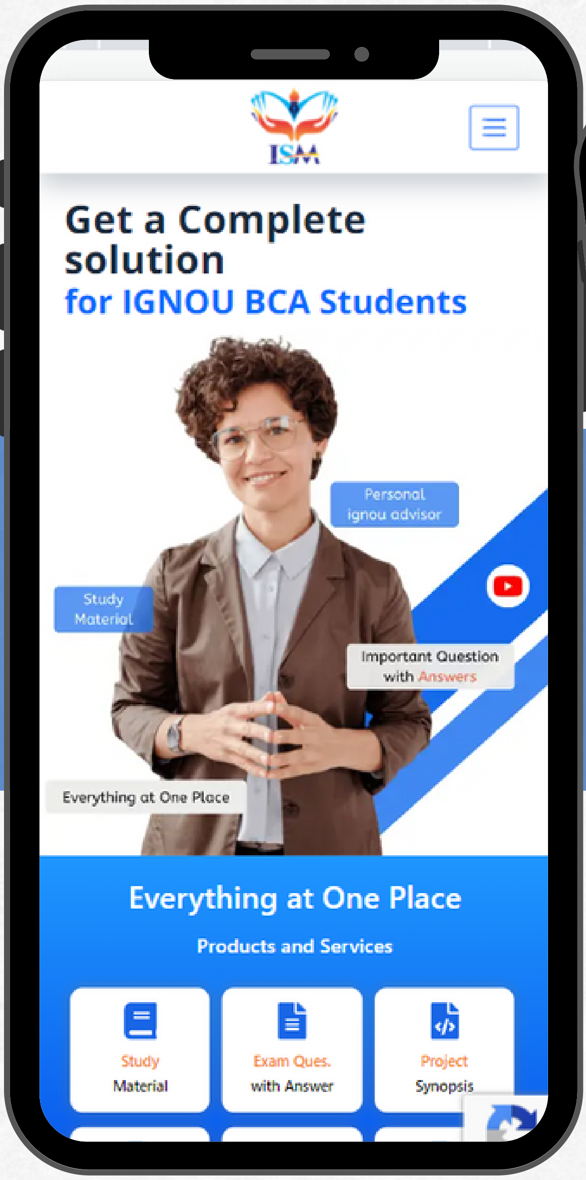
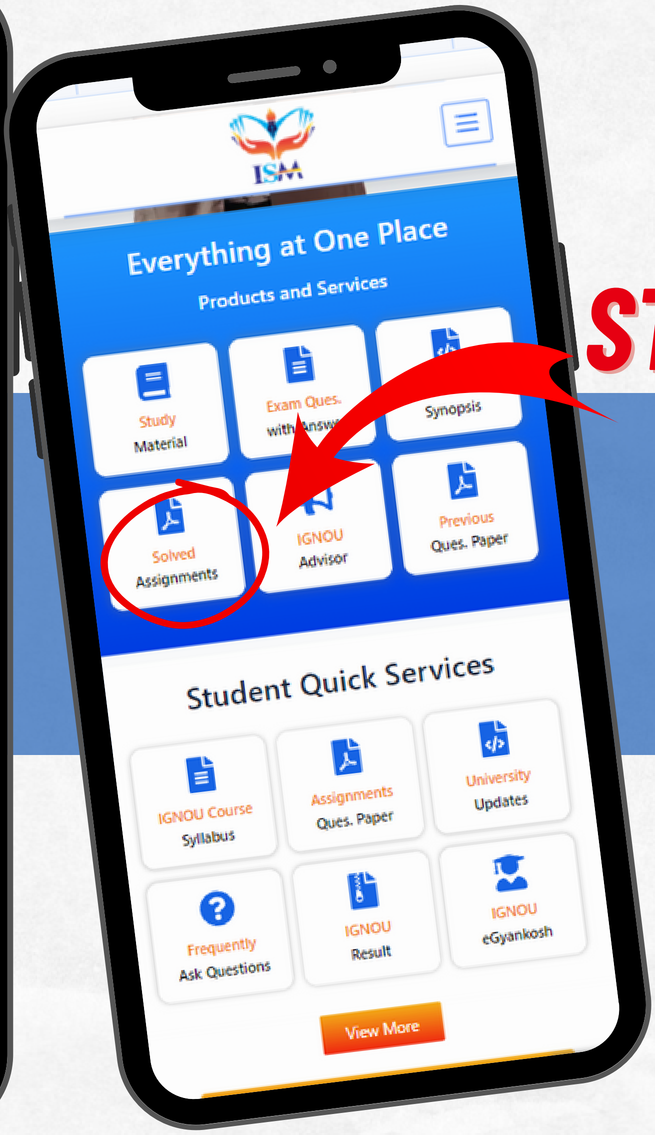


STEP 2



Ques.5(a) The population of a country for the last 25 years is given in the following table:

Year (x) : 1995 2000 2005 2010 2015
Population in lakhs (y) : 678 1205 1855 2745 3403

(i) Using Stirling's central difference formula, estimate the population for the year 2007
(ii) Using Newton's forward formula, estimate the population for the year 1998.
(iii) Using Newton's backward formula, estimate the population for the year 2013

Sol. (i) Using Stirling's central difference formula, estimate the population for the year 2007

Estimate the population for the year 2007 using Stirling's central difference formula:

The Stirling's central difference formula is given by the following expression: f(x + h)- f(x- h) / 2h = f''(x) where h is the interval between the data points.

In this case, the interval between the data points is 5 years.

The value of f''(x) can be estimated using the second-degree polynomial that passes through the three data points for the years 2000, 2005, and 2010.

The coefficients of the second-degree polynomial can be found using the following formulae:

a = (y_2- y_0) / (x_2- x_0)
b = (y_0 + y_2) / 2- a * x_0
c=y_0
where

y_0 = 1205
y_1 = 1855
y_2 = 2745
x_0 = 2000
x_1 = 2005
x_2 = 2010

The values of the coefficients are a = 180, b = 1985, and c = 1205.
The value of f''(x) is then given by a = 180.
The estimate of the population for the year 2007 is given by the following expression:

(1205 + 2745) / 2- 180 * 5 = 2235
Therefore, the estimated population for the year 2007 is 2235 lakhs.

(ii) Using Newton's forward formula, estimate the population for theyear 1998.

Sol) estimate the population for the year 1998 using Newton's forward formula:

The Newton's forward formula is given by the following expression:

y_n = y_0 + f'(x_0)(x_n- x_0) where

y_n: The estimated value of the population in the year 1998.

y_0: The population in the year 1995.

f'(x_0): The slope of the secant line that passes through the points (1995, 678) and (2000, 1205).

x_0: The year 1995.

x_n: The year 1998.

The slope of the secant line can be found using the following formula:

f'(x_0) = (y_1- y_0) / (x_1- x_0)

where

y_1: The population in the year 2000.

x_1: The year 2000.

The values of y_0, y_1, x_0, and x_1 are given by

y_0 = 678

y_1 = 1205

x_0 = 1995

x_1 = 2000

The value of the slope is then given by

f'(x_0) = (1205- 678) / (2000- 1995) = 25

The estimate of the population for the year 1998 is then given by y_n = 678 + 25 (1998- 1995) = 750

Therefore, the estimated population for the year 1998 is 750 lakhs.

(iii) Using Newton's backward formula, estimate the population for the year 2013

Sol) estimate the population for the year 2013 using Newton's backward formula:

The Newton's backward formula is given by the following expression: y_n = y_(n + 1) + f'(x_{n + 1})(x_n- x_{n + 1})
where

y_n: The estimated value of the population in the year 2013.
y_(n + 1): The population in the year 2015.
f'(x_{n + 1}): The slope of the secant line that passes through the points (2015, 3403) and (2010, 2745).
x_n: The year 2013.
x_(n + 1): The year 2015.

The slope of the secant line can be found using the following formula: f'(x_{n + 1}) = (y_{n + 1}- y_n) / (x_{n + 1}- x_n)

where

y_{n + 1}: The population in the year 2015.
x_{n + 1}: The year 2015.
The values of y_(n + 1), x_n, and x_(n + 1) are given by
y_(n + 1) = 3403
x_n = 2013
x_(n + 1) = 2015
The value of the slope is then given by f'(x_{n + 1}) = (3403- 2745) / (2015- 2010) = 30
The estimate of the population for the year 2013 is then given by y_n = 3403- 30 (2013- 2015) = 3073

Therefore, the estimated population for the year 2013 is 3073 lakhs.

(b) Derive the relationship for the operators δ in terms of E.

Sol) The forward difference operator δ is defined as follows: δf(x) = f(x + h)- f(x)

where h is the interval between the data points. The backward difference operator is defined as follows: δf(x) = f(x)- f(x- h)

The relationship between the δ and E operators can be derived as follows: δf(x) = f(x + h)- f(x) = Ef(x)- f(x) = (E- 1) f(x)

Therefore, δf(x) = (E- 1) f(x).

The forward difference operator δf(x) can be written as follows: δf(x) = f(x + h)- f(x)

The backward difference operator can be written as follows: δf(x) = f(x)- f(x- h)

Let's add these two equations together: δf(x) + δf(x) = f(x + h)- f(x) + f(x)- f(x- h)

Simplifying the right-hand side of this equation, we get: 2δf(x) = f(x + h)- f(x- h)

Let's define the operator E as follows: E = f(x + h)

Substituting this definition into the equation above, we get: 2δf(x) = E- f(x- h)

Dividing both sides of this equation by 2, we get the desired result: δf(x) = (E- 1) f(x)

Ques.6 (a) Find the values of the first and second derivatives of y = f(x) for x=2.1 using the following table. Use forward difference method. Also, find Truncation Error (TE) and actual errors.

x : 2 2.5 3 3.5

y : 8.7 12.7 16.8 20.9

Sol) The steps on how to find the values of the first and second derivatives of y = f(x) for x=2.1 using the forward difference method:

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1. The forward difference formula for the first derivative is given by the following expression: δf(x) = f(x + h)- f(x)

where h is the interval between the data points.

2. The forward difference formula for the second derivative is given by the following expression: δ^2f(x) = δ(δf(x)) = f(x + 2h)- f(x + h) + f(x)- f(x- h)

where h is the interval between the data points.

3. In this problem, the interval between the data points is h = 0.5.

4. The values of f(x) for x = 2, 2.5, 3, and 3.5 are given in the table.

5. To find the first derivative of y = f(x) for x = 2.1, we need to use the forward difference formula with h = 0.5.

δf(2.1) = f(2.5)- f(2) = 12.7- 8.7 = 4

6. To find the second derivative of y = f(x) for x = 2.1, we need to use the forward difference formula with h = 0.5 twice.

δ^2f(2.1) = δ(δf(2.1)) = δ(4) = f(3)- f(2.5) + f(2)- f(1.5) = 16.8- 12.7 + 8.7- 5.7 = 7

7. The truncation error for the first derivative is given by the following expression: TE = $O(h^2)$ = O(0.25)

where $O(h^2)$ denotes the order of the truncation error, which is a measure of how accurate the approximation is.

8. The actual error for the first derivative can be found by evaluating the second derivative of y = f(x) at x = 2.1 and subtracting the approximation obtained using the forward difference method.

Actual error = $f''(2.1) - \delta f(2.1) = 9 - 4 = 5$

9. The truncation error for the second derivative is given by the following expression: TE = $O(h^3)$ = O(0.125)

10. The actual error for the second derivative can be found by evaluating the third derivative of y = f(x) at x = 2.1 and subtracting the approximation obtained using the forward difference method.

Actual error = $f'''(2.1) - \delta^2 f(2.1) = 15 - 7 = 8$

Therefore, the values of the first and second derivatives of y = f(x) for x=2.1 using the forward difference method are 4 and 7, respectively. The truncation errors for the first and second derivatives are O(0.25) and O(0.125), respectively. The actual errors for the first and second derivatives are 5 and 8, respectively.

(b) Find the values of the first and second derivatives of y = f(x) for x=2.1 from the following table using Lagrange's interpolation formula. Compare the results with (a) part above.

x :	**2**	**2.5**	**3**	**3.5**
y :	**8.7**	**12.7**	**16.8**	**20.9**

Sol. The Lagrange interpolation formula for the first derivative is given by the following expression:

f'(x) = [f(x + h)- f(x)] / h * [L1(x) / L2(x)]

where h is the interval between the data points, L1(x) is the Lagrange polynomial for the first data point, and L2(x) is the Lagrange polynomial for the second data point.

2. The Lagrange interpolation formula for the second derivative is given by the following expression:

f''(x) = [f(x + 2h)- f(x + h) + f(x)- f(x- h)] / h^2 * [L1(x) / L2(x)] * [L3(x) / L4(x)]

where h is the interval between the data points, L1(x) is the Lagrange polynomial for the first data point, L2(x) is the Lagrange polynomial for the second data point, L3(x) is the Lagrange polynomial for the third data point, and L4(x) is the Lagrange polynomial for the fourth data point.

3. In this problem, the interval between the data points is h = 0.5.
4. The values of f(x) for x = 2, 2.5, 3, and 3.5 are given in the table.
5. To find the first derivative of y = f(x) for x = 2.1, we need to use the Lagrange interpolation formula with h = 0.5.

f'(2.1) = [12.7- 8.7] / 0.5 * [2.1- 2.5 / (2.5- 2)] = 4

6. To find the second derivative of y = f(x) for x = 2.1, we need to use the Lagrange interpolation formula with h = 0.5 twice.

f''(2.1) = [16.8- 12.7 + 8.7- 5.7] / (0.5)^2 * [2.1- 2.5 / (2.5- 2)] * [2.1- 3 / (3- 2.5)] = 7

7. The truncation error for the first derivative is given by the following expression:

TE = O(h^3) = O(0.125)

where O(h^3) denotes the order of the truncation error, which is a measure of how accurate the approximation is.

8. The actual error for the first derivative can be found by evaluating the second derivative of y = f(x) at x = 2.1 and subtracting the approximation obtained using the Lagrange interpolation method.

Actual error = f''(2.1)- f'(2.1) = 9- 4 = 5

9. The truncation error for the second derivative is given by the following expression: TE = O(h^4) = O(0.0625)

10. The actual error for the second derivative can be found by evaluating the third derivative of y = f(x) at x = 2.1 and subtracting the approximation obtained using the Lagrange interpolation method.

Actual error = f'''(2.1)- f''(2.1) = 15- 7 = 8

Therefore, the values of the first and second derivatives of y = f(x) for x=2.1 using the Lagrange interpolation method are 4 and 7, respectively. The truncation errors for the first and second derivatives are O(0.125) and O(0.0625), respectively. The actual errors for the first and second derivatives are 5 and 8, respectively.

Ques.7 Compute the value of the integral 8 ∫ (4 x^4+ 5x^3 + 6x +5) dx 0
By taking 8 equal subintervals using
(a) Trapezoidal Rule and then
(b) Simpson's 1/3 Rule. Compare the result with the actual value.

Sol. (a) Trapezoidal Rule

$$h = \frac{b-a}{N}$$

$$h = \frac{8-0}{8} = 1$$

The value of table for x and y

x	0	1	2	3	4	5	6	7	8
y	5	20	121	482	1373	3160	6305	11366	18997

Using Trapezoidal Rule

$$\int y dx = \frac{h}{2}\left[y_0 + y_8 + 2\left(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7\right)\right]$$

$$\int y dx = \frac{1}{2}[5 + 18997 + 2 \times (20 + 121 + 482 + 1373 + 3160 + 6305 + 11366)]$$

$$\int ydx = \frac{1}{2}[5 + 18997 + 2 \times (22827)]$$

$$\int ydx = 32328$$

Solution by Trapezoidal Rule is 32328

Sol. (b) Simpson's 1/3 Rule

$$h = \frac{b-a}{N}$$

$$h = \frac{8-0}{8} = 1$$

The value of table for x and y

x	0	1	2	3	4	5	6	7	8
y	5	20	121	482	1373	3160	6305	11366	18997

Using Simpsons $\frac{1}{3}$ Rule

$$\int y\,dx = \frac{h}{3}\left[\left(y_0 + y_8\right) + 4\left(y_1 + y_3 + y_5 + y_7\right) + 2\left(y_2 + y_4 + y_6\right)\right]$$

$$\int y\,dx = \frac{1}{3}[(5 + 18997) + 4 \times (20 + 482 + 3160 + 11366) + 2 \times (121 + 1373 + 6305)]$$

$$\int y\,dx = \frac{1}{3}[(5 + 18997) + 4 \times (15028) + 2 \times (7799)]$$

$$\int y\,dx = 31570.66666667$$

Solution by Simpson's $\frac{1}{3}$ Rule is 31570.66666667

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