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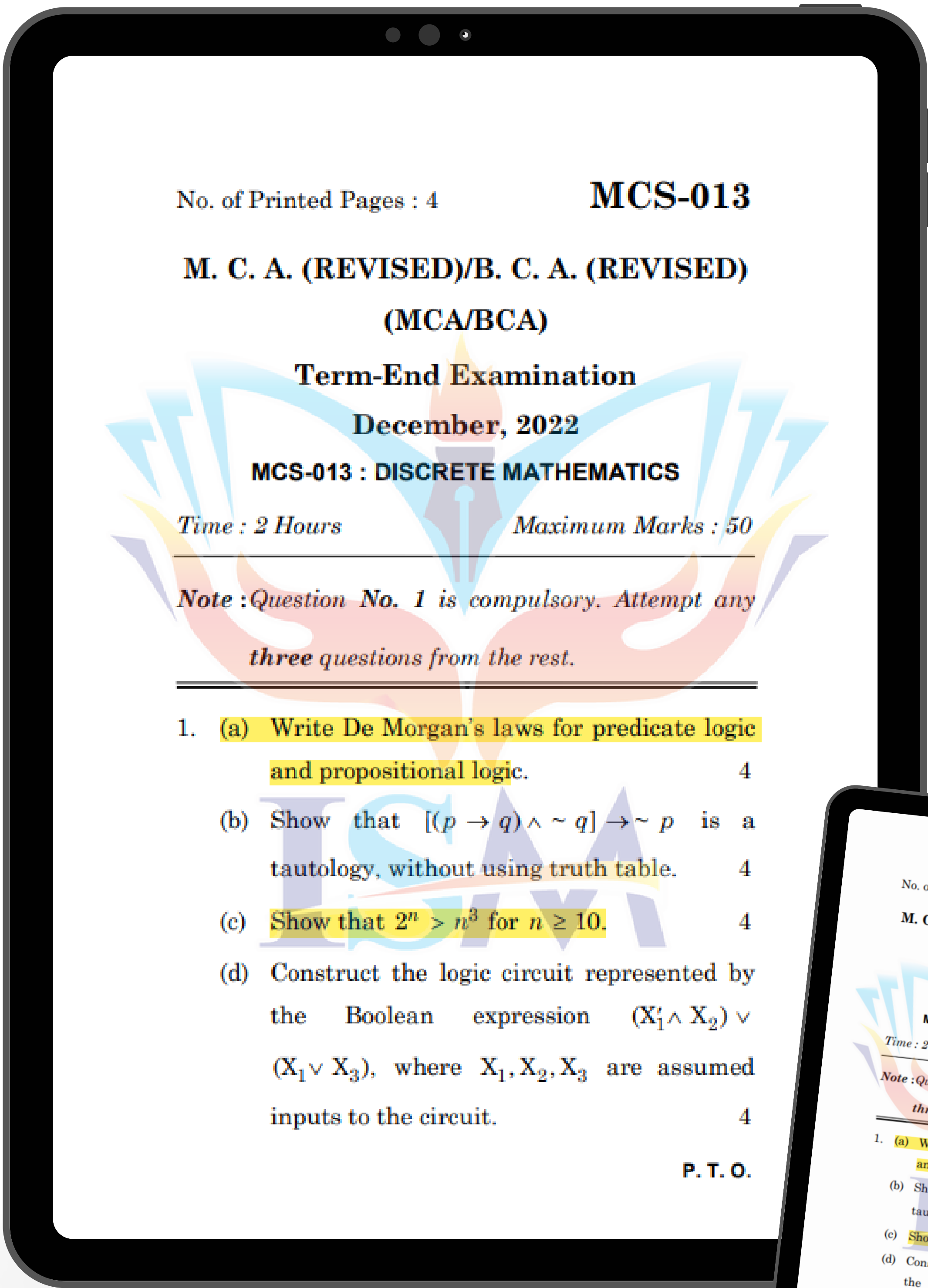
Previous year Question paper

with ANSWER's



No. of Printed Pages : 4 **MCS-013**

M. C. A. (REVISED)/B. C. A. (REVISED)

(MCA/BCA)

Term-End Examination

December, 2022

MCS-013 : DISCRETE MATHEMATICS

Time : 2 Hours *Maximum Marks : 50*

Note : *Question **No. 1** is compulsory. Attempt any **three** questions from the rest.*

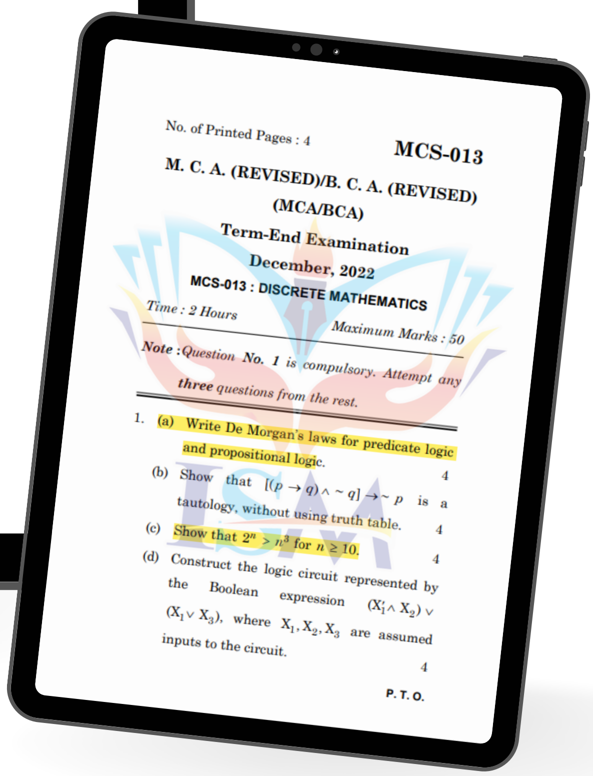
1. (a) Write De Morgan's laws for predicate logic and propositional logic. 4

(b) Show that $[(p \to q) \wedge \sim q] \to \sim p$ is a tautology, without using truth table. 4

(c) Show that $2^n > n^3$ for $n \geq 10$. 4

(d) Construct the logic circuit represented by the Boolean expression $(X_1' \wedge X_2) \vee (X_1 \vee X_3)$, where X_1, X_2, X_3 are assumed inputs to the circuit. 4

P. T. O.



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Ques. 1(b) Show that [( ) ~] ~ pq q p →∧ → is a tautology, without using truth table.

$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$

$\equiv [(\sim p \vee q) \wedge \sim q] \rightarrow \sim p$, using E18, and symmetricity of $\equiv$

. $\equiv [(\sim p \wedge \sim q) \vee (q \wedge \sim q)] \rightarrow \sim p$, by De Morgan's laws.

$\equiv [(\sim p \wedge \sim q) \vee F] \rightarrow \sim p$, since $q \wedge \sim q$ is always false.

$\equiv (\sim p \wedge \sim q) \rightarrow \sim p$,

Ques. 1(e) What is the difference between permutation and combination ? If n couples are at a dance party, in how many ways can the men and women be pained for a single dance ?

Permutation and combination are mathematical concepts used to count and arrange objects or events in different ways. The main difference lies in whether the order of the objects matters or not.

Permutation: A permutation refers to the arrangement of objects in a specific order. The order in which the objects are arranged matters. Permutations are used when you want to select and arrange objects where the order is important.

Combination: A combination refers to the selection of objects without considering the order. In combinations, the order of selection does not matter. Combinations are used when you want to select objects without regard to their order or arrangement.
In the case of a dance party with n couples, if you want to determine how many ways the men and women can be paired for a single dance, it is a combination because the order of pairing does not matter. Each couple will be dancing together regardless of the order in which they are chosen.

To calculate the number of combinations, you can use the formula for combinations:

C(n, k) = n! / (k!(n - k)!)

Where:

·n is the total number of couples

·k is the number of couples chosen to dance together in a single pair

·! denotes the factorial function, which means multiplying a number by all the positive integers less than it down to 1

For example, if there are 4 couples (n = 4), and you want to select 2 couples (k = 2) to dance together, the number of ways they can be paired can be calculated as:

C(4, 2) = 4! / (2!(4 - 2)!) = 24 / (2 * 2) = 6

Therefore, there are 6 different ways to pair the couples for a single dance.

Ques. 2(c) Show whether 15 is a rational or irrational.

Assume √15 is rational and can be expressed as a fraction a/b,
√15 = a/b
Squaring both sides of the equation:
15 = (a^2)/(b^2)
Cross-multiplying:
15 * (b^2) = (a^2)
This implies that a^2 is divisible by 15. Therefore, a must also be divisible by 15.
Let's rewrite a = 15k, where k is an integer.
Substituting this value back into the equation:
15 * (b^2) = (15k)^2 15 * (b^2) = 225k^2 b^2 = 15k^2

This shows that b^2 is also divisible by 15. Consequently, b must also be divisible by 15. Now, we have both a and b divisible by 15, which means they have a common factor. Therefore, our assumption that √15 is rational leads to a contradiction. Hence, √15 is not a rational number. Therefore, we can conclude that √15 is an irrational number.

Ques. 3(b) What is Cartesian product ? Give the geometric representation of the Cartesian product of A and B, where A = {2, 3, 4} and B = {1, 4}.

The Cartesian product is a mathematical operation that combines two sets to create a new set. It is denoted by the symbol "×" or simply a cross product. The resulting set contains all possible ordered pairs where the first element comes from the first set, and the second element comes from the second set.

Given A = {2, 3, 4} and B = {1, 4}, we can calculate their Cartesian product as follows:

A × B = {(2, 1), (2, 4), (3, 1), (3, 4), (4, 1), (4, 4)}

So, the Cartesian product of A and B is {(2, 1), (2, 4), (3, 1), (3, 4), (4, 1), (4, 4)}. It contains all possible combinations of elements from set A and set B.

Ques. 4(b) Determine all the integer solution to 1234 xxxx +++ = 9, where 1, i x ≥ i = 1, 2, 3, 4.

Determine all the integer solutions to the equation x1 + x2 + x3 + x4 = 9, where xi ≥ 1 for i = 1, 2, 3, 4, we can use a method called "stars and bars" or "balls and urns." This method allows us to distribute identical objects (in this case, the value of 9) into distinct boxes (in this case, the variables x1, x2, x3, and x4) while satisfying certain conditions.

Let's represent the equation as:

x1 + x2 + x3 + x4 = 9

We can introduce a new variable y1, where y1 = x1 - 1, y2 = x2 - 1, y3 = x3 - 1, and y4 = x4 - 1. Now we have:

y1 + y2 + y3 + y4 = 9 - 1 - 1 - 1 - 1 = 5

Since y1, y2, y3, and y4 can take any non-negative integer values, we can solve this equation using the stars and bars method.

Let's assume we have 5 stars (representing the value of 5) and 3 bars (representing the three "+" signs). The stars will be placed in between or at the ends of the bars to represent the values of y1, y2, y3, and y4. Each placement of stars and bars represents a unique solution to the equation.

For example, if we have:

||*|**

It represents y1 = 2, y2 = 2, y3 = 1, and y4 = 0.

Therefore, the corresponding solution for x1, x2, x3, and x4 would be:

x1 = y1 + 1 = 2 + 1 = 3 x2 = y2 + 1 = 2 + 1 = 3 x3 = y3 + 1 = 1 + 1 = 2 x4 = y4 + 1 = 0 + 1 = 1

So, one solution to the equation is x1 = 3, x2 = 3, x3 = 2, and x4 = 1.

By generating all possible arrangements of stars and bars, we can find all the integer solutions to the equation x1 + x2 + x3 + x4 = 9, where xi ≥ 1 for i = 1, 2, 3, 4.

Ques. 4(c) Prove by induction that 3 n n – is divisible by 3 for all positive integers.

Step 1 When n = 1: 1^3 - 3 = 1 - 3 = -2.

Since -2 is not divisible by 3, the statement does not hold true

Step 2: Induction Hypothesis Assume that for some positive integer k, the statement is true: k^3 - 3 is divisible by 3.

Step 3: Induction Step We need to prove that if the statement is true for k, then it must also be true for k + 1.

Consider (k + 1)^3 - 3: (k + 1)^3 - 3 = k^3 + 3k^2 + 3k + 1 - 3 = (k^3 - 3) + 3k^2 + 3k - 2.

From the induction hypothesis, we know that k^3 - 3 is divisible by 3. So, let's focus on the remaining terms: 3k^2 + 3k - 2.

We can rewrite this expression as: 3(k^2 + k) - 2.

Since k^2 + k is an integer (as k is a positive integer), we can represent it as a single integer, let's say m.

Therefore, (k + 1)^3 - 3 = 3m - 2.

Now, we need to show that 3m - 2 is divisible by 3.
We can express 3m - 2 as 3m - 3 + 1.
This can be further simplified as 3(m - 1) + 1.
Since m - 1 is an integer (as m is an integer), we can represent it as another integer, let's say p.
Therefore, 3m - 2 = 3p + 1.
We have shown that (k + 1)^3 - 3 can be expressed in the form 3p + 1, where p is an integer. This implies that (k + 1)^3 - 3 is not divisible by 3.
Based on the induction step, we have shown that if the statement holds true for k, it does not hold true for k + 1.
Therefore, we can conclude that the statement "n^3 - 3 is divisible by 3 for all positive integers" is not true.

Ques. 5(b) Write the principle of duality. Find the dual of :

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Consider (k + 1)^3 - 3: (k + 1)^3 - 3 = k^3 + 3k^2 + 3k + 1 - 3 = (k^3 - 3) + 3k^2 + 3k - 2.
From the induction hypothesis, we know that k^3 - 3 is divisible by 3. So, let's focus on the remaining terms: 3k^2 + 3k - 2.
We can rewrite this expression as: 3(k^2 + k) - 2.
Since k^2 + k is an integer (as k is a positive integer), we can represent it as a single integer, let's say m.
Therefore, (k + 1)^3 - 3 = 3m - 2.

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