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Previous year Question paper

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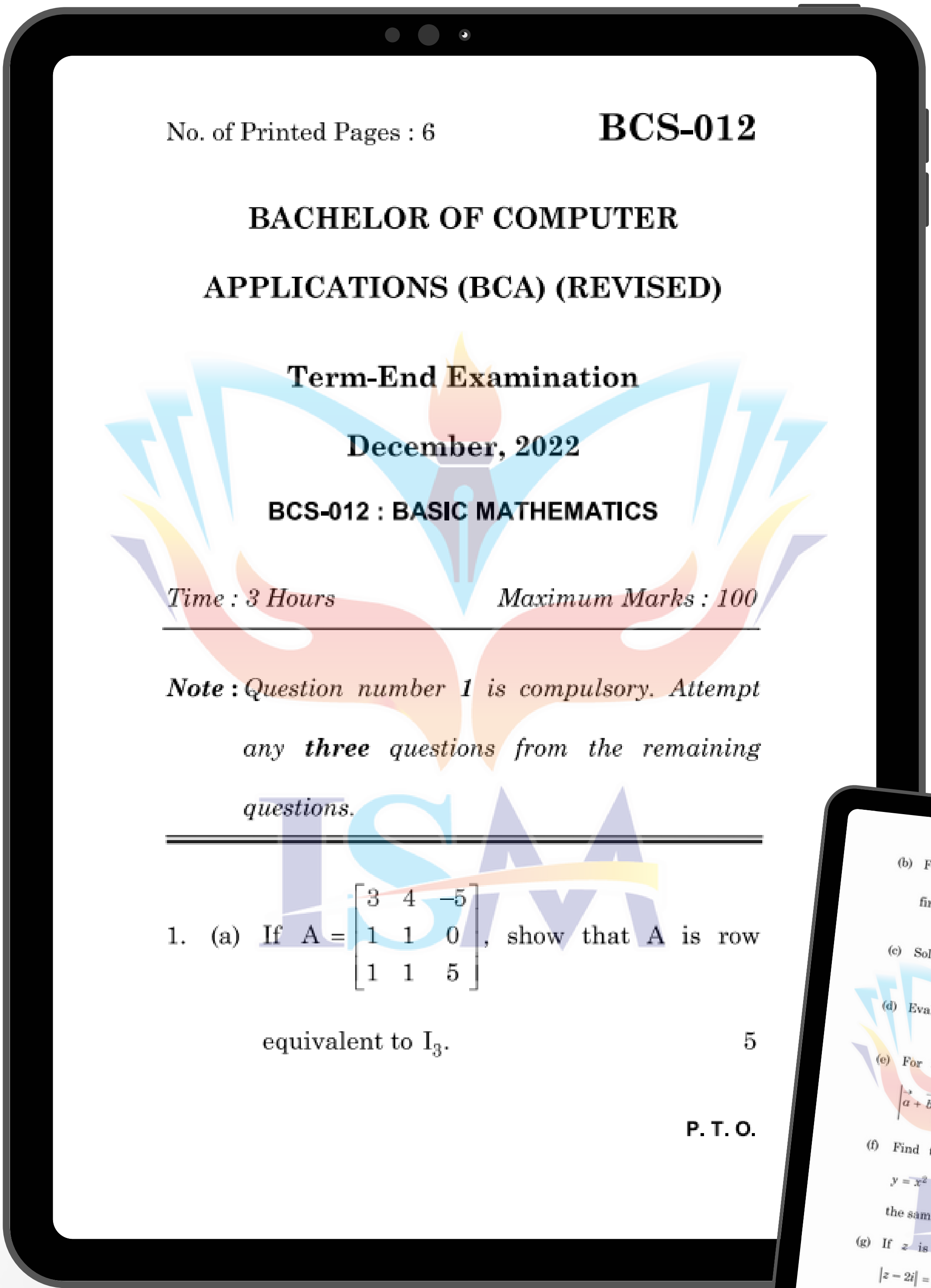
with ANSWER's



No. of Printed Pages : 6 **BCS-012**

BACHELOR OF COMPUTER APPLICATIONS (BCA) (REVISED)

Term-End Examination

December, 2022

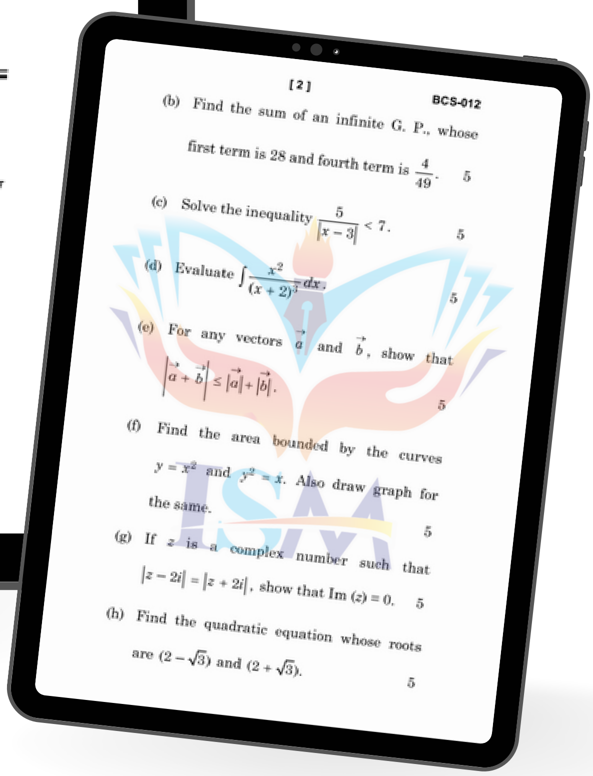
BCS-012 : BASIC MATHEMATICS

Time : 3 Hours *Maximum Marks : 100*

Note : *Question number **1** is compulsory. Attempt any **three** questions from the remaining questions.*

1. (a) If $A = \begin{bmatrix} 3 & 4 & -5 \\ 1 & 1 & 0 \\ 1 & 1 & 5 \end{bmatrix}$, show that A is row equivalent to I_3. 5

P. T. O.



[2] BCS-012

(b) Find the sum of an infinite G. P., whose first term is 28 and fourth term is $\frac{4}{49}$. 5

(c) Solve the inequality $\frac{5}{|x-3|} < 7$. 5

(d) Evaluate $\int \frac{x^2}{(x+2)^3} dx$. 5

(e) For any vectors $\vec{a}$ and $\vec{b}$, show that $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$. 5

(f) Find the area bounded by the curves $y = x^2$ and $y^2 = x$. Also draw graph for the same. 5

(g) If z is a complex number such that $|z - 2i| = |z + 2i|$, show that Im $(z) = 0$. 5

(h) Find the quadratic equation whose roots are $(2 - \sqrt{3})$ and $(2 + \sqrt{3})$. 5

Ques. 1(d) Show that [( ) ~] ~ pq q p →∧ → is a tautology, without using truth table.

Q1 d) $\int \frac{x^2}{(x+2)^3} dx$

By Substitution method

let $x+2 = t$

$dx = dt$

$I = \int \frac{(t-2)^2}{(t)^3} dt$

$= \int \frac{t^2 - 4t + 4}{t^3} dt$

$= \int \frac{t^2}{t^3} dt - 4\int \frac{t}{t^3} dt + 4\int \frac{1}{t^3} dt$

$= \int \frac{1}{t} dt - 4\int t^{-2} dt + 4\int t^{-3} dt$

$= \log|t| - 4\frac{t^{-2+1}}{-2+1} + 4\frac{t^{-3+1}}{-3+1} + C$

$= \log|t| + 4t^{-1} - 2t^{-2} + C$

$= \log|t| + \frac{4}{t} - \frac{2}{t^2} + C$

$\log|x+2| + \frac{4}{x+2} - \frac{2}{(x+2)^2} + C$

Ques. 1(e) For any vectors $\vec{a}$ and $\vec{b}$, show that

$$\left|\vec{a} + \vec{b}\right| \leq \left|\vec{a}\right| + \left|\vec{b}\right|.$$ 5

$$|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$$

Solution : If $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then the inequality holds trivially.

So let $|\vec{a}| \neq 0 \neq |\vec{b}|$. Then,

$$|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b})^2 = (\vec{a} + \vec{b}).(\vec{a} + \vec{b})$$

$$= \vec{a}\vec{a} + \vec{a}\vec{b} + \vec{b}\vec{a} + \vec{b}\vec{b}$$

$$= |\vec{a}|^2 + 2\vec{a}\vec{b} + |\vec{b}|^2 \quad (\because \ \vec{a}\vec{b} = \vec{a}\vec{b})$$

$$= |\vec{a}|^2 + 2|\vec{a}||\vec{b}|\cos\theta + |\vec{b}|^2 \text{ where } \theta \text{ is the angle between } \vec{a} \text{ and } \vec{b}$$

$$\leq |\vec{a}|^2 + 2|\vec{a}||\vec{b}| + |\vec{b}|^2 \quad (\because \cos\theta \leq 1 \ \forall \ \theta)$$

$$= (|\vec{a}| + |\vec{b}|)^2$$

Hence $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$

Ques. 1(f) Find the area bounded by the curves y=x² and y² =x also draw graph for the same.

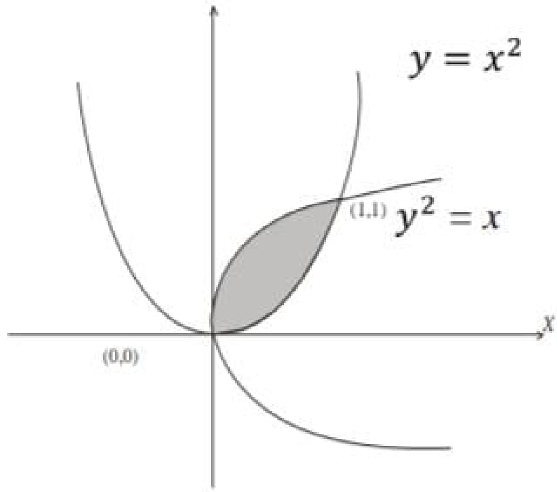
4. We first find the points of intersection of $y = x^2$ and $y^2 = x$. We have
$x = y^2 = (x^2)^2$

$\Rightarrow x = x^4$

$\Rightarrow x(1 - x^3) = 0$

$\Rightarrow x = 0$ or $x = 1$

Required area



$$= \int_0^1 (\sqrt{x} - x^2)\, dx = \left[\frac{x^{3/2}}{\frac{3}{2}} - \frac{x^3}{3} \right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \text{ sq. units}$$

Ques. 1(g) If z is a complex number such that $|z - 2i| = |z + 2i|$, show that $\text{Im}(z) = 0$. 5

$|z - 2i| = |z + 2i|$

Now we will substitute the value of z in the given equation.

$$|x + iy - 2i| = |x + iy + 2i|$$
$$|x + i(y - 2)| = |x + i(y + 2)|$$
$$x^2 + (y - 2)^2 = x^2 + (y + 2)^2$$
$$(y - 2)^2 = (y + 2)^2$$
$$y^2 + 4 - 4y = y^2 + 4 + 4y$$
$$-4y = 4y$$
$$8y = 0$$
$$y = 0$$
$$ie$$
$$Img(z) = 0$$

Hence we proved that

$Img(z) = 0$

Ques. 1(h) Find the quadratic equation whose roots are $(2 - \sqrt{3})$ and $(2 + \sqrt{3})$. 5

1) h)

Root 1 = $(2-\sqrt{3})$
Root 2 = $(2+\sqrt{3})$

Equation : x^2 − (Sum of roots)x + (product of roots) = 0

Sum of roots = $2 - \sqrt{3} + 2 + \sqrt{3}$
= 4

Product = $(2-\sqrt{3})(2+\sqrt{3})$
= 4 − 3
= 1

Eqn :

$x^2 - 4x + 1 = 0$

Ques. 2(a) (a) Show that : 5

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (y-x)(z-x)(z-y)$$

Q2 a)

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (y-x)(z-x)(z-y)$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$$

Taking Common

Ques. 2(a)

$$= (y-x)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix}$$

$R_3 \to R_3 - R_2$

$$= (y-x)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 0 & z-y \end{vmatrix}$$

Taking common

$$= (y-x)(z-x)(z-y)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding:

$$\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & y+x \\ 0 & 1 \end{vmatrix} - x\begin{vmatrix} 0 & y+x \\ 0 & 1 \end{vmatrix} + x^2\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$= 1(1-(0)) - x(0) + x^2(0)$$

$$= (1)$$

$$= (y-x)(z-x)(z-y)$$

Proved

Ques. 3(b)

$a = 22, d = -4$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2(22) + (n-1)(-4)].$$

$$= \frac{n(4)}{2}[11 - n + 1] = 2n(12 - n)$$

Now, $2n(12 - n) = 64$

$\Rightarrow \quad n^2 - 12n + 32 = 0$

$\Rightarrow \quad (n-4)(n-8) = 0$

$\Rightarrow \quad n = 4, n = 8$

Ques. 4(c)

Solution: Let P_n denote the statement

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

When $n = 1, Pn$ becomes $\frac{1}{1.2}=\frac{1}{1+1}$ or $\frac{1}{2}=\frac{1}{2}$

This shows that the result holds for $n = 1$. Assume that P_k is true for some $k \in \mathbf{N}$.

That, is assume that

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+\frac{1}{k(k+1)}=\frac{k}{k+1} \qquad (1)$$

We shall now show that the true of P_k implies the truth of P_{k+1} where P_{k+1} is

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots+\frac{1}{k(k+1)}=\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$$

$$\text{LHS of (1)} = \frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}$$

$$=\frac{k}{(k+1)}+\frac{1}{(k+1)(k+2)} \qquad \text{[induction assumption]}$$

$$=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{k+1}{k+2}$$

$= \text{RHS of (1)}$

This shows that the result holds for $n = k+1$; therefore, the truth of P_k implies the truth of P_{k+1}. The two steps required for a proof by mathematical induction have been completed, so our statement is true for each natural number n.

Ques. 4(d)

Solution : $A = \begin{bmatrix} 5 & 3 & 8 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

Applying $R_1 \leftrightarrow R_3$, we have

$$A \sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 5 & 3 & 8 \end{bmatrix}$$

Applying $R_3 \to R_3 - 5R_1$, we have

$$A \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 8 & 8 \end{bmatrix}$$

Applying elementary row operations $R_1 \to R_1 + R_2$ and $R_3 \to R_3 - 8R_2$, we have

$$A \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, we apply elementary column operation $C_3 \to C_3 - C_2$, to get

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Again, applying $C_3 \to C_3 - C_1$, we have

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Ques. 4(d)

Rank :

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$

$= 0$

Determinant $= 0$

Thus, we Reduce the matrix in 2×2

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$= 1$

Determinant $= 1$

then, Rank $= 2$ (size of matrix)

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